Lecture 19

Application to Business and Economics

Let $C(x)$ be the total cost that a company incurs in producing x units of a certain commodity. The function $C(x)$, is called a cost function.

The rate of change of cost with respect to the number of units produced is called the marginal cost. Therefore, $C'(x) \equiv$ marginal cost

Let $p(x)$ be the price per unit that the company can charge if it sells x units. Then p is called the demand function (or price function)

If x units are sold and price per unit is $p(x)$, then the total revenue is

$R(x) = x \cdot p(x)$, and R is called the revenue function.

The derivative R' of the revenue function is called the marginal revenue function.

If x units are sold, then the total profit is $P(x) = R(x) - C(x)$ and P is called the profit function.

The marginal profit function is P'

Ex A store has been selling 200 Tablets a week at \$ 350 each.

A market survey indicates that for each \$ 10 rebate offered to buyers, the number of units sold will increase by 20 a week

Find the demand function and the revenue function. How large a rebate should the store offer to maximize it's revenue ?

Solution If x is the number of tablets sold per week, then the weekly increase in sales is $x - 200$.

For each increase of 20 units sold, the price is decreased by \$ 10. So for each additional unit sold, the decrease in price will be $\frac{1}{20} \cdot 10$ and therefore the demand function is

$$p(x) = 350 - \frac{1}{2}(x - 200) = 450 - \frac{1}{2}x$$

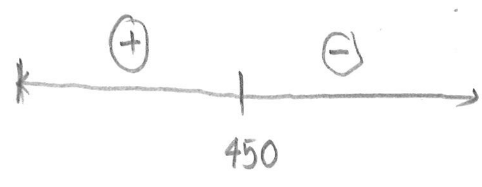
Then the revenue function is

$$R(x) = x\,p(x) = x\left(450 - \frac{1}{2}x\right) = 450x - \frac{1}{2}x^2$$

Then $R'(x) = 450 - x$

So, $R'(x) = 0 \Rightarrow x = 450$

So max revenue when $x = 450$.



Then the corresponding price is

$$p(450) = 450 - \frac{1}{2}(450) = 225$$

and therefore the rebate is $350 - 225 = \underline{125}$. ☒

<u>Ex</u>. If $C(x) = 16{,}000 + 500x - 1.6x^2 + 0.004x^3$

is the cost function and $p(x) = 1700 - 7x$ is the demand function, find the production level that will maximize profit.

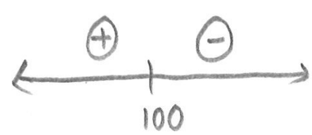
<u>Sol^n</u> $R(x) = xp(x) = x(1700 - 7x) = 1700x - 7x^2$

Then the profit function, $P(x) = R(x) - C(x)$

$$= 1700x - 7x^2 - (16000 + 500x - 1.6x^2 + 0.004x^3)$$

$$= 1200x - 5.4x^2 - 0.004x^3 - 1600.$$

To find maximum profit, $P'(x) = 0 \Leftrightarrow$



$$1200 - 10.8x - 0.012x^2 = 0$$

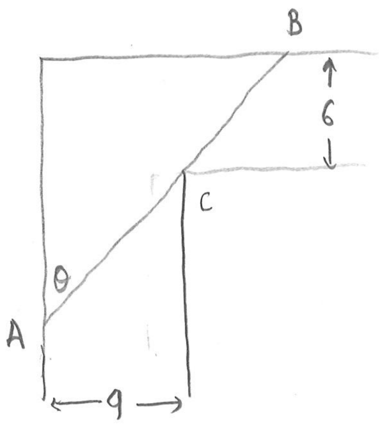
$$x^2 + 900x - 100{,}000 = 0$$

$$(x + 1000)(x - 100) = 0$$

$$x = 100$$

So maximum profit when $x = 100$.

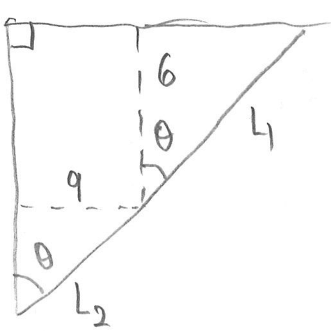
Ex A steel pipe is being carried down a hallway 9 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner?



Paradoxically, we will find the maximum length by solving a minimum problem

Let L be the length of the line ACB going from wall to wall touching the corner C.

If $\theta \to 0$, or $\theta \to \frac{\pi}{2}$, we have $L \to \infty$ as L can be as large as we want, and so there will be an angle θ that will make the length minimum, and a pipe of this length will just fit around the corner.



$$L = L_1 + L_2$$

$$= 9\operatorname{cosec}\theta + 6\sec\theta$$

$$\frac{dL}{d\theta} = -9\operatorname{cosec}\theta \cdot \cot\theta + 6\sec\theta \cdot \tan\theta$$

$$\frac{dL}{d\theta} = 0 \Rightarrow 6\sec\theta . \tan\theta = 9\operatorname{cosec}\theta . \cot\theta$$

not concerned about $\theta = 0, \frac{\pi}{2}$ so
$\Rightarrow$

$$\frac{\sec\theta . \tan\theta}{\operatorname{cosec}\theta . \cot\theta} = \frac{9}{6} \Rightarrow \tan^3\theta = \frac{3}{2} \Rightarrow \tan\theta = \sqrt[3]{1.5} \approx 48.86^\circ$$

$\Updownarrow$

0.853^c

$\ominus$ $\oplus$

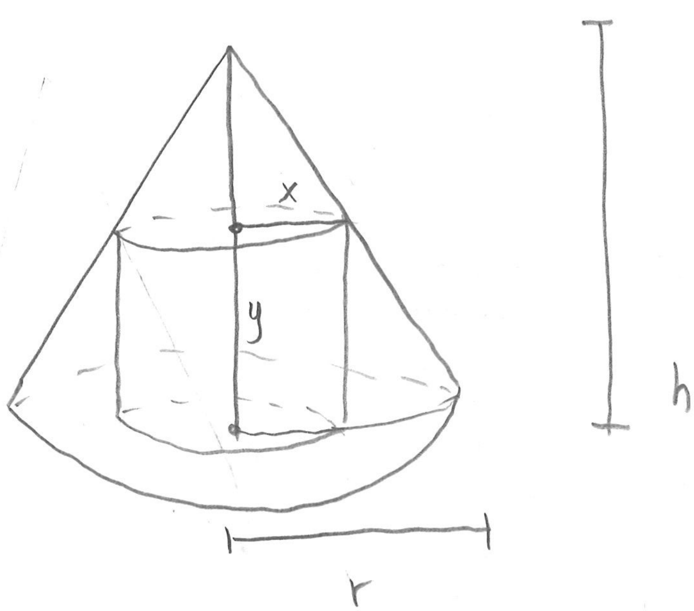
0° — 48.86° — 90°

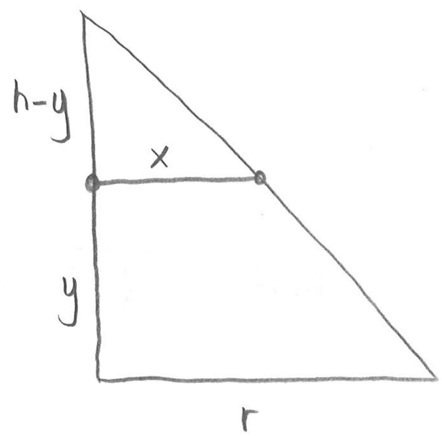
So min.

So the longest Length of pipe $L = 9(\operatorname{cosec} 48.6^\circ) + 6\sec(48.6)^\circ$

≈ 21.07 ft

Ex A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.





So, $\frac{h}{r} = \frac{h-y}{x}$

$xh = r(h-y) \Rightarrow \frac{xh}{r} = h-y$

$y = h - \frac{xh}{r}$

Then $V = \pi x^2 y = \pi x^2\left(h - \frac{xh}{r}\right) = -\frac{\pi x^3 h}{r} + \pi x^2 h$

Then $V'(x) = -\frac{3\pi x^2 h}{r} + 2\pi x h$, $0 \leq x \leq r$

Then, $V'(x) = 0 \Rightarrow 2\pi x h - \frac{3\pi x^2 h}{r} = 0 \Rightarrow \pi x h\left(2 - \frac{3x}{r}\right) = 0$

$\Rightarrow x = 0$ or $x = \frac{2r}{3}$

$V(0) = 0$

$V(r) = \pi r^2 h - \frac{\pi r^3 h}{r} = \pi r^2 h - \pi r^2 h = 0$

$V\left(\frac{2}{3}r\right) = \pi\left(\frac{2}{3}r\right)^2 h - \frac{\pi\left(\frac{2}{3}r\right)^3 h}{r}$

$= \frac{4\pi r^2 h}{9} - \frac{8\pi r^2 h}{27} = \frac{4\pi r^2 h}{27}$